Charmonium-Hadron Cross Section in a Nonperturbative QCD Approach

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Abstract

We calculate the nonperturbative $J/\Psi - N$ and $\Psi' - N$ cross sections with the model of the stochastic vacuum which has been succesfully applied in many high energy reactions. We also give a quantitative discussion of pre-resonance formation and medium effects.
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During the next year the first data on heavy ion collisions at high energy ($\sqrt{s} = 200$ GeV per nucleon pair) will be available at RHIC. As it is well known, one of the main goals of this machine is to find and study the plasma of quarks and gluons (QGP) [1,2]. The search for this state of matter has started in the early eighties and since then has been subject of intense debate [1,2]. Recent results on Pb-Pb collisions, taken at lower energies at CERN-SPS, have attracted a great attention and increased the hope that a new phase of nuclear matter is "just around the corner".

The signature of QGP formation has been and still remains a theoretical and experimental challenge. Indeed there is so far no "crucial test" able to disentangle the possible new phase from the dense hadronic background. Among the proposed signatures the most

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interesting is the suppression of J/Ψ [3]. This suppression was observed experimentally by the NA38 collaboration in 1987 in collisions with light ions and also and more dramatically by the NA50 collaboration in 1995-1996 in Pb-Pb collisions [4,5].

Whereas the old data could be reasonably well explained by a "conventional" approach the new Pb-Pb results of the NA50 collaboration created a big controversy [6]. The entire set of the J/Ψ data from pA and AB collisions available before the advent of the Pb beam at CERN SPS has been found to be consistent with the nuclear absorption model. However, in the case of the new Pb-Pb data, the density of secondaries, which (together with primary nucleons flying around) are presumably responsible for the charmonium absorption, is so high that the hadronic system in question is hardly in a hadronic phase. Nevertheless, a conventional treatment of the problem is not yet discarded [7].

Reliable values for the charmonium nucleon cross sections are of crucial importance in the present context. One needs to know the cross section $\sigma_{J/\psi}$ in order to predict a nuclear suppression of J/Ψ without assuming a so called "deconfining regime". Estimates using perturbative QCD give values which are too small to explain the observed absorption conventionally, but they are certainly not reliable for that genuine nonperturbative problem. A nonperturbative estimate may be tried by applying vector dominance to J/Ψ and Ψ' photoproduction. In this way a cross section of $\sigma_{J/\psi} \simeq 1.3$ mb for $\sqrt{s} \simeq 10$ GeV and $\sigma_{\psi'}/\sigma_{J/\psi} \simeq 0.8$ has been obtained [8,9]. In ref. [10] strong arguments against the vector dominance with only few intermediate vector mesons were put forward even in the case of the production of light vector mesons and which apply a fortiori to the production of heavy vector mesons. The instability of the vector dominance model can be seen from a more refined multichannel analysis [8] where a value $\sigma_{J/\psi} \simeq 3-4$ mb has been obtained. Even this value is too small in order to explain the absorption in p-A collisions which is of the order $\sigma_{\psi}^{abs} \simeq 7.3$ mb [11,12]

These hadron-hadron cross sections involve nonperturbative aspects of QCD dynamics and therefore require a nonperturbative model to be calculated. In a recent letter [13] the nonperturbative QCD contribution to the charmonium-nucleon cross section was evaluated by using an interpolation formula for the dependence of the cross section on the transverse size of a quark-gluon configuration. In this work we calculate the J/Ψ and Ψ' - nucleon cross sections in a specific nonperturbative model of QCD: the model of the stochastic vacuum (MSV) [14–17]. It has been applied to a large number of hadronic and photoproduction processes (including photoproduction of J/Ψ) with remarkably good success. Its application to J/Ψ and Ψ' nucleon scatttering is straightforward. We also investigate the influence of the nuclear matter and arrive at rather stringent limits for the cross sections in an environment different from the vacuum and where the properties of the medium are reflected in a shift of the J/Ψ mass [18].

The basis of the MSV is the calculation of the scattering amplitude of two colourless dipoles [10,17] based on a semiclassical treatment developped by Nachtmann [19]. For details we refer to the literature and show here only some intermediate steps necessary for the understanding of the text. The dipole-dipole scattering amplitude is expressed as the expectation value of two Wegner-Wilson loops with lightlike sides and transversal extensions $\vec{r}_{t1}$ and $\vec{r}_{t2}$ respectively. This leads to a profile function $J(\vec{b}, \vec{r}_{t1}, \vec{r}_{t2})$ from which hadron-hadron scattering amplitudes are obtained by integrating over different dipole sizes with the transversal densities of the hadrons as weight functions according to

$$\sigma^{tot}_{J/\Psi} = \int d^2b\, d^2r_{t1}\, d^2r_{t2}\, \rho_{J/\Psi}(\vec{r}_{t1})\, \rho_N(\vec{r}_{t2})\, J(\vec{b}, \vec{r}_{t1}, \vec{r}_{t2}) \ . \qquad (1)$$

Here $\rho_{J/\Psi}(\vec{r}_{t1})$ and $\rho_N(\vec{r}_{t2})$ are the transverse densities of the J/Ψ and nucleon respectively.

The basic ingredient of the model is the gauge invariant correlator of two gluon field strength tensors. The latter is characterized by two constants: the value at zero distance, the gluon condensate $< g^2FF >$, and the correlation length a. We take these values from previous applications of the model [10] (and literature quoted there):

$$< g^2FF >= 2.49\mathrm{GeV}^4 \qquad \mathrm{a} = 0.346\mathrm{fm} \ . \qquad (2)$$

The wave functions of the proton have been determined from proton-proton and proton-antiproton scattering respectively. It turns out that the best description for the nucleon transverse density is given by that of a quark diquark system with transversal distance $\vec{r}_t$ and density:

$$\rho_N(\vec{r}_t) = |\Psi_p(\vec{r}_t)|^2 = \frac{1}{2\pi}\frac{1}{S_p^2}\, e^{-\frac{|\vec{r}_t|^2}{2S_p^2}} \ . \qquad (3)$$

The value of the extension parameter, $S_p = 0.739$ fm, obtained from proton-proton scattering agrees very well with that obtained from the electromagnetic form factor in a similar treatment [20].

For the wave function of the J/Ψ we used two approaches:
1) A numerical solution of the Schroedinger equation with the standard Cornell potential [21]:

$$V = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r \ . \qquad (4)$$

2) A Gaussian wave function determined by the electromagnetic decay width of the J/Ψ which has been used in a previous investigation of J/Ψ photoproduction [10].

For the Ψ' no analysis of photoproduction in the model has been made so we use only the solution of the Schroedinger equation.

The linear potential can be calculated in the model of the stochastic vacuum which yields the string tension:

$$\sigma = \frac{8\kappa}{81\pi} < g^2FF > a^2 \ = 0.179\,\mathrm{GeV}^2 \ , \qquad (5)$$

where the parameter κ has been detemined in lattice calculations to be $\kappa = 0.8$ [22].

The other parameters, the charmed (constituent) mass and the (frozen) strong coupling can be adjusted in order to give the correct J/Ψ and Ψ' mass difference and the J/ψ decay width

$$m_c = 1.7\mathrm{GeV} \qquad \alpha_\mathrm{s} = 0.39 \ . \qquad (6)$$

We also use the standard Cornell model parameters [21]:

$$\alpha_s = 0.39, \sigma = 0.183 \ \mathrm{GeV}^2 \text{ and } m_c = 1.84 \ \mathrm{GeV}. \qquad (7)$$

From the numerical solution $\psi(|\vec{r}\,|)$ of the Schroedinger equation the transversal density is projected:

$$\rho_{J/\Psi}(\vec{r}_t) = \int \left|\psi(\sqrt{\vec{r}_t^{\,2} + r_3^2})\right|^2 dr_3 \;, \tag{8}$$

where $\vec{r}_t$ is the J/Ψ transversal radius.

Given the values of α_s, σ and m_c we solve the non-relativistic Schroedinger equation numerically, obtain the wave function, compute the transverse wave function and plugg it into the MSV calculation [17]. The results are shown in Table I. In this table $\sqrt{< r^2 >}$ is the root of the mean square distance of quark and antiquark and $\sqrt{< r_t^2 >}$ is the root of the mean square transversal distance of quark and antiquark. Wave function A) is the one obtained with the parameters given by Eqs. (5) and (6). Wave function B) corresponds to the standard Cornell model parameters [21] Eq. (7).

Wave function	$\sqrt{< r^2 >}$ fm	$\sqrt{< r_t^2 >}$ fm	σ_{tot} [mb]
$J/\Psi(1S)$			
A	0.393	0.321	4.48
B	0.375	0.306	4.06
C			4.69
$\Psi(2S)$			
A:	0.788	0.640	17.9

TABLE I $J/\Psi - N$ and $\Psi' - N$ cross section. A and B: numerical solution of the Schroedinger equation with parameters in Eqs. (6) and (7) respectively. C: Cross section obtained by the weighted average of the longitudinally and transversely polarized J/Ψ wave functions of ref. [10].

In ref. [10] a gaussian ansatz was made to construct vector meson wave functions that describe well the electromagnetic decay of the vector meson and photo and electroproduction cross sections. In Table I, wave function C) gives the result for the $J/\Psi - N$ cross section obtained with the weighted average of the longitudinally and transversely polarized J/Ψ wave functions from [10] with transversal sizes $\sqrt{< r_t^2 >} = 0.327$ fm and 0.466 fm.

Averaging over our results for different wave functions, our final result for the $J/\Psi - N$ cross section is

$$\sigma_{J/\psi} = 4.4 \pm 0.6\,\text{mb} \;. \tag{9}$$

The error is an estimate of uncertainties coming from the wave function and the model. The only other nonperturbative calculation of the $J/\Psi - N$ cross section that we are aware of was done in ref. [13] and the obtained cross section was $\sigma_{J/\psi} = 3.6$ mb, in a fair agreement with our result and with recent analysis of J/Ψ photoproduction data [8]. For Ψ' our cross section is also of the same order as the value obtained in [13]: $\sigma_{\psi'} = 20.0$ mb.

Since one of the possible explanations of the observed J/Ψ suppression is based on the pre-resonance absorption model [11] we present numerical calculations of the nucleon - pre-resonant charmonium state cross section, σ_ψ. In the pre-resonance absorption model, the pre-resonant charmonium state is either interpreted as a color-octet, $(c\bar{c})_8$, and a gluon in the hybrid $(c\bar{c})_8 - g$ state, or as a coherent $J/\Psi - \Psi'$ mixture. We use a gaussian transverse wave function, as in Eq. (3), to represent a state with transversal radius $\sqrt{< r_t^2 >} \simeq 0.82 \sqrt{< r^2 >} = \sqrt{2} S_\psi$ (S_ψ is the pre-resonance extension parameter analogous to S_p). With the knowledge of the wave functions and transformation properties of the constituents we can compute the total cross section given by the MSV. The resulting nucleon - pre-resonant charmonium state cross section will be different if the pre-resonant charmonium state consists of entities in the adjoint representation (as $(c\bar{c})_8 - g$) or in the fundamental representation (as a $J/\Psi - \Psi'$ mixture), the relation being $\sigma_{\text{adjoint}} = \frac{2N_C^2}{N_C^2-1}\sigma_{\text{fundamental}}$, with $N_C = 3$. In Table II we show the results for these two possibilities and different values of the transverse radius.

$\sqrt{< r_T^2 >}$ (fm)	$\sigma_{c\bar{c}}$ (mb)	$\sigma_{(c\bar{c})_8-g}$ (mb)
0.20	1.79	4.02
0.25	2.76	6.21
0.30	3.96	8.91
0.35	5.30	11.92
0.40	6.81	15.32
0.45	8.50	19.12
0.50	10.28	23.13

TABLE II The cross section charmonium-nucleon for gaussian wave-functions and different values of the transverse radius ($\sqrt{< r_t^2 >}$) of the $c\bar{c}$ in a singlet state (first row) or in a hybrid $(c\bar{c})_8 - g$ state (second row).

From our results we can see that a cross-section $\sigma_\psi^{abs} \simeq 6 - 7$ mb, needed to explain the J/Ψ and Ψ' suppression in p-A collisions in the pre-resonance absorption model [11,7], is consistent with a pre-resonant charmonium state of size $\simeq 0.50 - 0.55$ fm if it is a $J/\Psi - \Psi'$ mixture or $\simeq 0.30 - 0.35$ fm for a $(c\bar{c})_8 - g$ state.

So far the calculations were done with the vacuum values of the correlation length and gluon condensate generally used in the MSV [17]. However, since the interaction between the charmonium and the nucleon occurs in a hadronic medium, these values may change. Indeed, lattice calculations [22,23] show that both the correlation length and the gluon condensate tend to decrease in a dense (or hot) medium. The reduction of the string tension, σ, leads to two competing effects, which can be quantitatively compared in the MSV. On one hand the cross section tends to decrease strongly when the gluon condensate or the correlation length decrease. On the other hand, when the string tension is reduced the $c - \bar{c}$ state becomes less confined and will have a larger radius, which, in turn, would lead to a larger cross section for interactions with the nucleons in the medium. It is of major interest to determine which of these effects is dominant.

The dependence of the total cross section, Eq. (1), on the extension parameters S_p and S_ψ is quite well parametrized [24] as:

$$\sigma_{J/\psi} \propto < g^2FF >^2 a^{10} \left(\frac{S_p}{a}\right)^{1.5} \left(\frac{S_\psi}{a}\right)^2 \tag{10}$$

In the MSV the string tension, σ, is related to the gluon condensate and to the correlation length through Eq. (5). Therefore, the dependence of the cross section on the string tension and correlation length is approximately given by:

$$\sigma_\psi \propto \sigma^2 a^6 \left(\frac{S_p}{a}\right)^{1.5} \left(\frac{S_\psi}{a}\right)^2 . \tag{11}$$

In a rough approximation the hadron radii can be estimated, using the Ritz variational principle, to be:

$$S \propto \left(\frac{1}{\sigma}\right)^{1/3} , \tag{12}$$

and thus we finally obtain the following three possibilities to express the cross section as a function of the string tension, σ, the correlation length, a, and the gluon condensate, $< g^2FF >$:

$$\sigma_{\psi N} \propto \begin{cases} \sigma^{5/6} a^{5/2} \\ \sigma^{25/12} < g^2FF >^{-5/4} \\ < g^2FF >^{5/6} a^{25/6} \end{cases} \tag{13}$$

From the equation above we see that the final effect of the medium is a reduction in the cross section. We can also see that a 10% variation in the parameters lead to large variations on the cross sections. Using the values of the correlation length and the gluon condensate reduced by 10%: $a = 0.31$ fm , $\langle g^2FF \rangle = 2.25$ GeV^4, we obtain a 40% reduction in the cross sections. Taking this reduction into account the cross sections obtained in this work are smaller than the ones needed (both in Refs. [11] and [7]) to explain experimental data. However, since in our model the nucleon - pre-resonant charmonium state cross section is much bigger than the $J/\Psi - N$ cross section if the pre-resonance charmonium state is a hybrid $(c\bar{c})_8 - g$ state, the reduction in the cross sections due to medium effects favors the pre-resonant model for the hadronic explanation of the observed J/Ψ suppression.

In order to get more precise results we have varied only one the parameters a and $< g_s^2FF >$ and kept the other fixed. This was done in such a way as to decrease the string tension according to equation (5) to the values given in Table III, first row. The numerically evaluated values for the mass of the J/Ψ (serving as a physical measure of the change) and the cross sections are given in rows 2 to 4, using Gaussian wave functions determined by the variational principle.

string tension [GeV^2] (GeV^2)	ΔE [MeV] MeV	σ_{tot} [mb] (mb) a const.	σ_{tot} [mb] (mb) $< g^2FF >$ const.
σ_0	0	4.48	4.48
$0.9\sigma_0$	-33	3.86	3.38
$0.75\sigma_0$	- 83	3.27	2.2
$0.50\sigma_0$	-174	2.0	0.83
$0.25\sigma_0$	-275	0.91	0.13

TABLE III J/Ψ-Nucleon total cross sections as function of the string tension with either the correlation length or the gluon condensate kept constant. ΔE is the mass decrease of the J/Ψ due to the change of string tension.

To summarize, we calculated the nonperturbative $J/\Psi - N$ and $\Psi' - N$ cross sections with the MSV. The basic ingredient of the model is the gauge invariant correlator of two gluon field strength tensors which is characterized by two constants: the gluon condensate and the gluon field correlation length. Using for these quantities values fixed in previous applications and using well accepted charmonium wave functions we obtain $\sigma_{J/\psi\, N} \sim 4$ mb and $\sigma_{\psi'\, N} \sim 18$ mb. An interesting prediction of the MSV is the strong depedence on the parameters of the QCD vacuum which will most likely lead to a drastic reduction of them at higher temperatures and perhaps also at higher densities.

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